Stress Analysis of Stepped-Lap Joints with Bondline Flaws

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A stress analysis technique has been developed for load transfer in metal-to-composite adhesively bonded step-lap joints with bondline flaws. The problem is formulated based on the assumptions that both the metal and the composite are under generalized plane stress condition and that the adhesive acts as a shear spring. A set of differential equations is obtained for the load transferred from the metallic layer to the composite. Two types of bondline flaws are considered: debonds in the adhesive and gaps between step ends. The differential equations together with the appropriate boundary conditions are solved for each case. Numerical results are presented for force and nominal stress in the composite layer, stress in the metal, and shear stress in the adhesive.

Introduction

ADHESIVE bonded joints have features that make them attractive for advanced composite-to-metal joining, which is the reason for their current use in several important aircraft structural applications. Among the most significant concerns regarding the structural reliability of such types of bonded joints are the possible delaminations occurring in the composite layer and the debonding taking place in the adhesive during fatigue loads.

The fatigue failure of a bonded joint with or without initial flaws may be due to the failure of either the adhesive or the adherends. During cyclic loading, a flaw may develop (if not pre-existing) in the adhesive, thus causing local debonding that will grow under cyclic loading. The propagation of these debonds in the adhesive joint will depend on the stress level and stress concentration around the debond. As a result of the debonding and consequent local load redistributions, a flaw may also initiate in the adherend and propagate. The flaw may initiate in the adhesive or adherend at any of the steps of a step-lap joint and propagate. The resulting added load transfer to the composite may initiate failure in the composite. In order to predict the life of such joints, it is necessary to know the stresses in joints with such flaws.

The stress analysis methods for bonded joints, using mathematical and finite element techniques, have been discussed by various investigators; some of these methods are discussed in Refs. 1-17. The majority of the analytical work reported in the literature deals with adhesive joints without bondline flaws. The analysis of adhesive joints with flaws has been carried out by Hart-Smith[18] but the analysis does not consider end gaps. Mathematical and finite element methods of analysis have been used in these references. In all of the analytical work on composite-to-metal bonded joints reported in literature, the composite is treated as an anisotropic or orthotropic material.

In the present investigation the linear analysis of joints is considered. The analytical techniques developed by Erdogan and Ratwani[1] have been extended to cases where the joints have bondline flaws and end gaps at the steps.

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Formulation of Problem

General Formulation

Consider the stepped-lap joint shown in Fig. 1, where an isotropic plate, 1, is adhesively bonded to an orthotropic plate, 2, through a stepped-lap joint. For the following formulation, subscripts *1, 2,* and *3* refer to plate 1, plate 2, and the adhesive, respectively. The thickness of the adhesive layer at each step is uniform but may be different for each step. Let $\sigma_{1x}(x)$ and $\sigma_{2x}(x)$ be the stresses in the plates and $\tau(x)$ the shear stress in the adhesive on the interface under a uniform tensile force p_0 applied to the plates away from the joint. The problem is formulated under the following assumptions[1]:

1) The thicknesses h_1, h_{1i}, h_2, h_{2i}, and h_{3i} are small compared to the other dimensions of the structure, so that the individual layers may be under generalized plane stress (i.e., $\sigma_{1y}=0=\sigma_{2y}$).

2) The thickness variation of the stresses in the plates will be neglected under the usual assumption that the surface shear transmitted through the adhesive layer acts as a body force.

3) In the z direction (see Fig. 1), it will be assumed that $\bar{\epsilon}_z=\epsilon_{1z}=\epsilon_{2z}=0$. For the ith portion of the stepped joint (i.e., $\ell_i<x<\ell_{i+1}$), let h_{1i}, h_{2i}, and h_{3i} be the thicknesses, $p_{1i}(x)$ and $p_{2i}(x)$ the resultant forces per unit width acting in the x direction, $u_{1i}(x)$ and $u_{2i}(x)$ the displacements in the x direction, and $\tau_i(x)$ the adhesive shear stress. The adhesive forces acting on the $x=\ell_i$ parts of the steps (such as AB in Fig. 1) are neglected. From the equilibrium of the plate 2 and the adhesive layer we obtain

$$p_{2i}(x)=p_{2i-1}(\ell_{i-1})+\int_{\ell_{i-1}}^{x}\tau_i(x)\,\mathrm{d}x, \quad (i=1,\ldots,n) \tag{1}$$

$$\tau_i(x)=\frac{G_3}{h_{3i}}(u_{2i}-u_{1i}), \quad (i=1,\ldots,n) \tag{2}$$

where G_3 is the shear modulus of the adhesive. It is assumed that the material of plate 1 is isotropic with constants E_1 and ν_1, and plate 2 is orthotropic with constants E_{2x}, ν_{2x}, ν_{2z}, and G_2.

From assumption 3, $\bar{\epsilon}_z=\epsilon_{1z}=\epsilon_{2z}=0$, we have $\sigma_{1z}=\nu_1\sigma_{1x}$ and $\sigma_{2z}=\nu_{2z}\sigma_{2x}$ and, from assumption 1, $\sigma_{1y}=\sigma_{2y}=0$; therefore, the forces and stresses in plates 1 and 2 can be written as

$$p_{1i}=p_0-p_{2i}, \quad \sigma_{1x}=\frac{p_{1i}}{h_{1i}}, \quad \sigma_{2x}=\frac{p_{2i}}{h_{2i}} \tag{3}$$

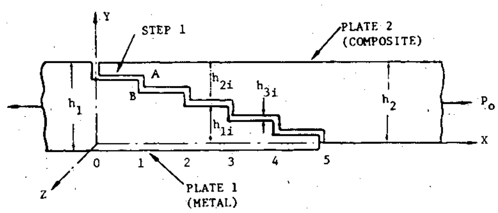


Fig. 1 Metal-to-composite adhesively bonded stepped-lap joint.

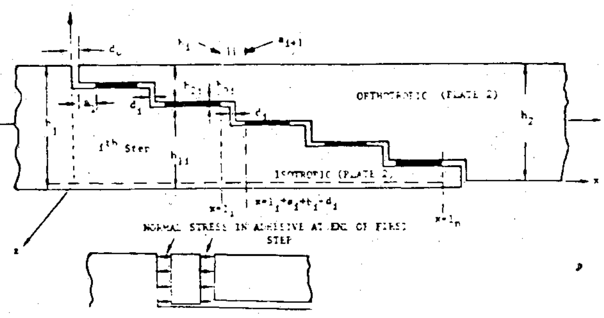


Fig. 2 Stepped-lap joint with bondline flaws.

The stress-strain relations then give

$$\epsilon_{1x}(x) = \frac{1-\nu_1^2}{E_1 h_{1i}} [p_0 - p_{2i}(x)]$$

$$\epsilon_{2x}(x) = \frac{1-\nu_{2x}^i \nu_{2z}^i}{E_{2x}^i h_{2i}} p_{2i}(x) \tag{4}$$

The strain-displacement relations are given by

$$\epsilon_{1x} = \frac{\mathrm{d}u_{1i}}{\mathrm{d}x}, \quad \epsilon_{2x} = \frac{\mathrm{d}u_{2i}}{\mathrm{d}x} \tag{5}$$

From Eqs. (1), (2), and (4) the equilibrium condition of the ith step of plate 2 is

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} p_{2i} - \alpha_i^2 p_{2i} = \beta_i p_0, \quad (i=1,...,n) \tag{6}$$

where n is the number of steps and

$$\alpha_i^2 = \frac{G_3}{h_{3i}} \left[\frac{1-\nu_1^2}{E_1 h_{1i}} + \frac{1-\nu_{2x}^i \nu_{2z}^i}{E_{2x}^i h_{2i}} \right], \quad \beta_i = -\frac{G_3}{h_{3i}} \frac{1-\nu_1^2}{E_1 h_{1i}} \tag{7}$$

Boundary Conditions

The solution of the differential equation (6) may be written as

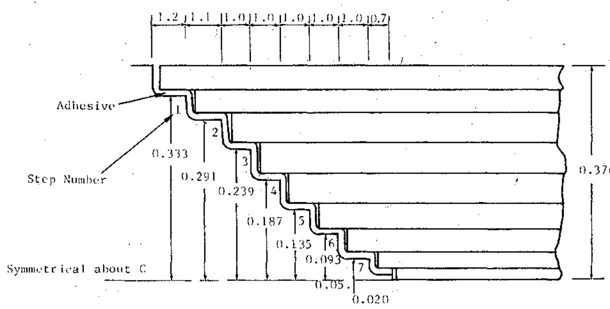
$$p_{2i}(x) = A_i e^{-\alpha_i x} + B_i e^{\alpha_i x} - \frac{\beta_i p_0}{\alpha_i^2}, \quad (i=1,...,n) \tag{8}$$

The $2n$ integration constants A_i and B_i are determined from the appropriate boundary conditions. These conditions can be obtained by considering the nature of the bondline flaws. Two types of bondline flaws are considered: debonds in the adhesive layer and gaps between step ends.

Joints with Debonds in the Adhesive Layer

Consider the stepped-lap joint with bondline flaws (such as a_1) shown in Fig. 2. The flaws are assumed to be at the end of the steps. The differential equations (6) are applicable to the joint geometry. The boundary conditions at the ends are

$$p_{21}(a_1) = 0 \text{ and } p_{2n}(\ell_n) = p_0 \tag{9}$$



ISOTROPIC YOUNGS MODULUS E1 = 0.16000E+08   POISSON RATIO NU = 0.310
ADHESIVE SHEAR MODULUS G3 = 0.57000E+05
COMPOSITE PROPERTY

STEP	E_{2x}	E_{2z}	ν_{2x}	ν_{2z}
1	0.10890E+08	0.35510E+07	0.64130	0.20911
2	0.10890E+08	0.35510E+07	0.64130	0.20911
3	0.11510E+08	0.34660E+07	0.62850	0.18926
4	0.11780E+08	0.34240E+07	0.62250	0.18094
5	0.11940E+08	0.34000E+07	0.61890	0.17624
6	0.11670E+08	0.34420E+07	0.62200	0.18094
7	0.11670E+08	0.34420E+07	0.62500	0.18434
8	0.11360E+08	0.34870E+07	0.63160	0.19387

Fig. 3 Eight-step joint used in numerical example.

For additional boundary conditions, consider the displacements in plates 1 and 2.

At $x = \ell_i$,

$$\tau(\ell_i) = \frac{\mathrm{d}}{\mathrm{d}x} p_{2i}(\ell_i) = \frac{G_3}{h_{3i}} [u_2(\ell_i) - u_1(\ell_i)] \tag{10}$$

At $x = \ell_i + a_{i+1} + b_i$,

$$\tau(\ell_i + a_{i+1} + b_i) = \frac{\mathrm{d}}{\mathrm{d}x} p_{2,i+1}(\ell_i + a_{i+1} + b_i)$$

$$= \frac{G_3}{h_{3,i+1}} [u_2(\ell_i + a_{i+1} + b_i) - u_1(\ell_i + a_{i+1} + b_i)]$$

$$= \frac{G_3}{h_{3,i+1}} [u_2(\ell_i) + \Delta u_2 - u_2(\ell_i) - \Delta u_1] \tag{11}$$

where Δu_1 and Δu_2 are the incremental displacements of plates 1 and 2 between ℓ_i and $\ell_i + a_{i+1} + b_i$. Combining Eqs. (9) and (10) the displacement boundary conditions become

$$\frac{\mathrm{d}}{\mathrm{d}x} p_{2,i+1}(\ell_i + a_{i+1} + b_i) = \frac{h_{3i}}{h_{3,i+1}} \frac{\mathrm{d}}{\mathrm{d}x} p_{2i}(\ell_i) + \Delta\tau_i$$
$$(i=1,2,...,n-1) \tag{12}$$

where

$$\Delta\tau_i = G_3 / h_{3,i+1} (\Delta u_2 - \Delta u_1)$$

$$\Delta u_2 = \epsilon_2^1 b_i + \epsilon_2^2 a_{i+1}$$

$$\Delta u_1 = \epsilon_1^1 (b_i - d_i) + \epsilon_1^2 (a_{i+1} + d_i) \tag{13}$$

where ϵ_2^1 is the strain in plate 2 at ℓ_i; ϵ_2^2 the strain in plate 2 at $\ell_i + b_i + a_{i+1}$; ϵ_1^1 the strain in plate 1 at ℓ_i; and ϵ_1^2 the strain in plate 1 at $\ell_i + b_i + a_{i+1}$.

Another set of boundary conditions is obtained from the consideration that there is no load transfer between $x = \ell_i$ and $x = \ell_i + a_{i+1} + b_i$. Thus

$$p_{2i}(\ell_i) = p_{2,i+1}(\ell_i + a_{i+1} + b_i) \quad (i=1,2,...,n-1) \tag{14}$$

Equations (8), (11), and (13) provide $2n$ conditions for the integration constants A_i and B_i in Eq. (7).

Joints with Bondline Flaws and End Gaps

During the fabrication process, gaps sometimes occur at the end of a joint due to the composite layer not butting against the metallic part. These gaps are generally filled with

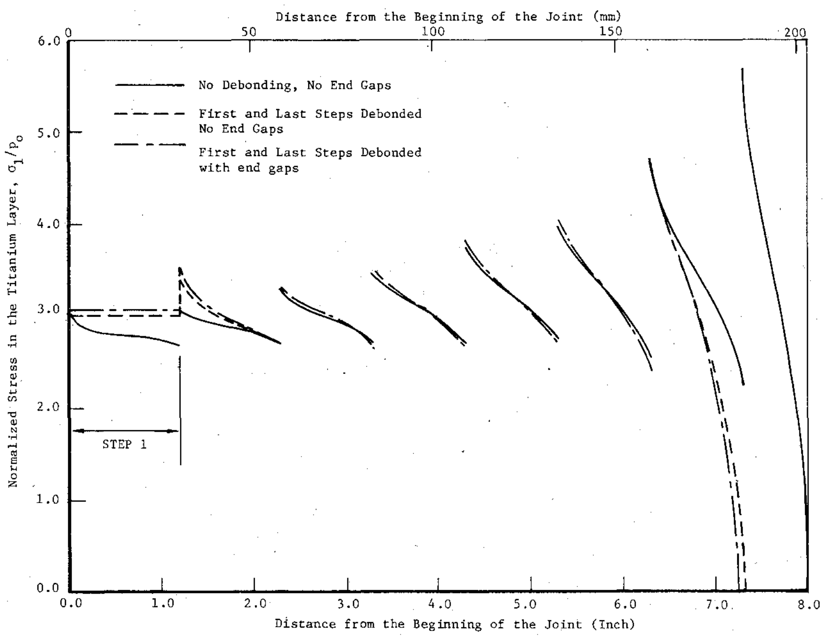


Fig. 4 Stresses in the titanium layer of the eight-step joint.

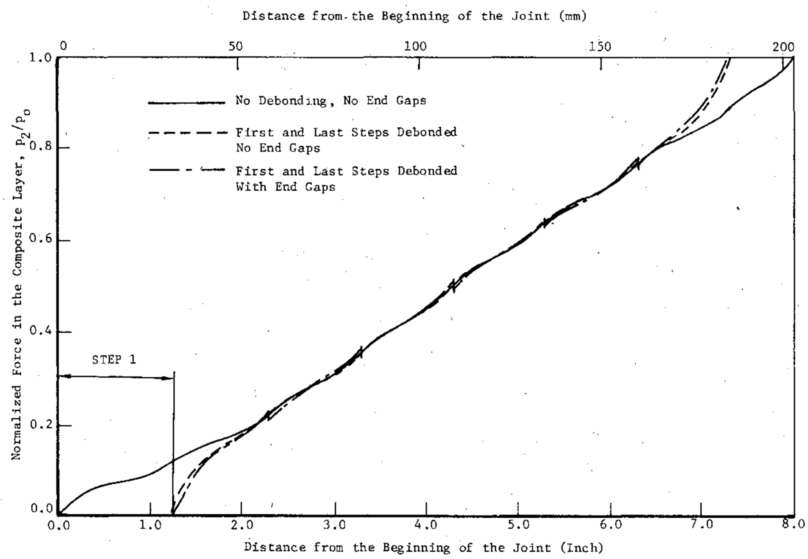


Fig. 5 Forces in the composite layer of the eight-step joint.

adhesive, with the result that there can be an axial force transfer between the metallic and composite layers through the adhesive.

The analysis of the stepped-lap joint with bondline flaws and end gaps is carried out on the assumption that the adhesive layer in the end gaps acts as a tension spring. Defining the total contact loads in the x direction acting on the steps $x=\ell_i (i=0,1,\ldots,n)$ by k_i, the equilibrium condition is given as

$$p_{2i}(x) = p_{2i-1}(\ell_{i-1}) + \int_{\ell_{i-1}}^{x} \tau_i(\zeta)\,\mathrm{d}\zeta + k_{i-1}, \quad (i=1,\ldots,n) \tag{15}$$

The differential equation (6) is valid with the same coefficients α_i and β_i. However, there are $n+1$ new unknown constants k_i. The additional conditions to determine these constants are obtained by considering the equilibrium of the adhesive layers at $x=\ell_i$ $(i=0,1,\ldots,n)$. The conditions are given as

$$\epsilon_3 = \frac{(1-\nu_3^2)k_i}{E_3(h_{2,i+1}-h_{2i})} = \frac{u_2(\ell_i+b_i+d_i)-u_1(\ell_i+b_i)}{d_i} \tag{16}$$

where E_3 is the Young's modulus of the adhesive. Thus, the boundary conditions for this problem are given as

$$p_{21}(d_0) = k_0, p_{2n}(\ell_n) = p_0 - k_n$$

$$p_2(\ell_i+b_i+d_i+a_{i+1}) = p_2(\ell_i) + k_i \quad (i=1,2,\ldots,n-1)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}p_2(\ell_i+b_i+d_i+a_{i+1}) = \frac{\mathrm{d}}{\mathrm{d}x}p_2(\ell_i) \quad (i=1,2,\ldots,n-1)$$

$$\frac{(1-\nu_3^2)}{E_3(h_{2,i+1}-h_{2i})}k_i = \frac{u_2(\ell_i+b_i+d_i)-u_1(\ell_i+b_i)}{d_i} \quad (i=1,2,\ldots,n) \tag{17}$$

Numerical Examples

A computer program has been developed to solve for the coefficients A_i and B_i in Eq. (8) numerically with the respective boundary conditions for the two types of flaws. The program computes the force in plate 2 (composite), the shear stress in the adhesive, and the extensional stresses in plates 1 (metallic) and 2. In addition, for the cases with the step end gaps, the end-gap forces are also computed.

An eight-step titanium-to-composite joint is chosen as an example. The joint configuration is shown in Fig. 3. The total

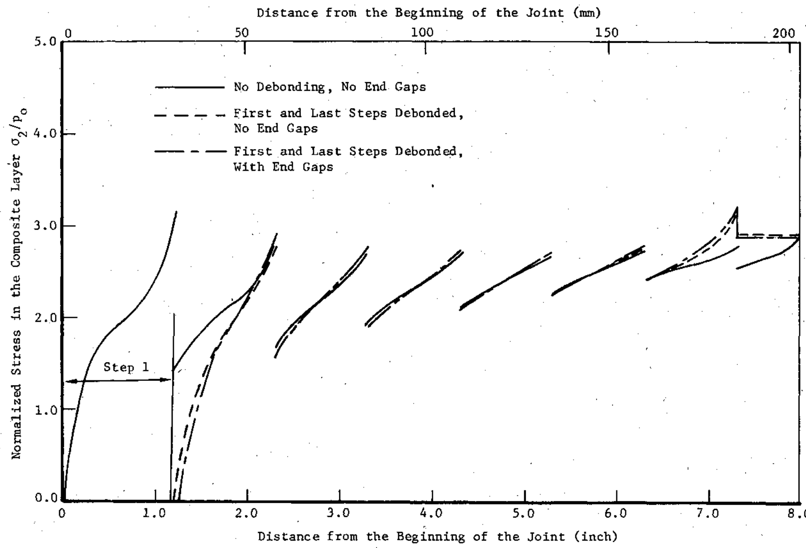


Fig. 6 Stresses in the composite layer of the eight-step joint.

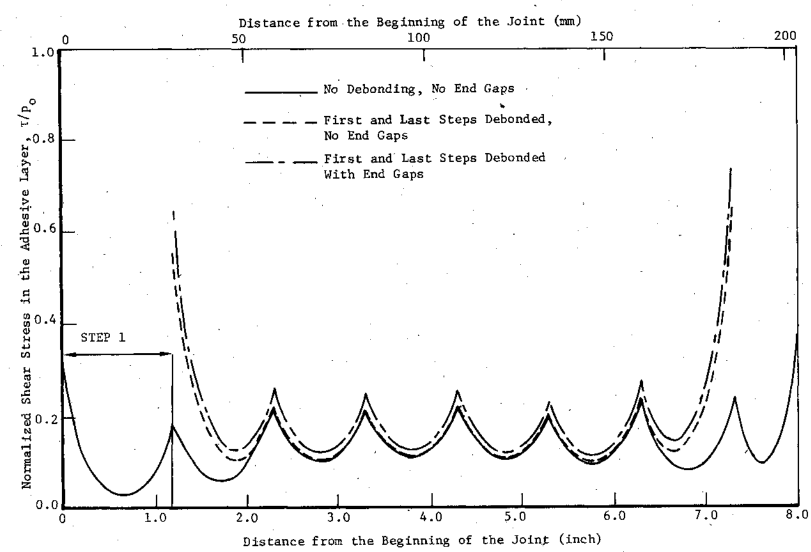


Fig. 7 Shear stresses in the adhesive layer of the eight-step joint.

length of the joining area is 8.0 in. (203.2 mm); the half-thickness of the joint away from the joining area is 0.376 in. (9.55 mm). The material properties of the titanium, the composite, and the adhesive are also given in Fig. 3. Three cases are analyzed: 1) no bondline flaws, 2) the first and the last steps completely debonded but no step end gaps, and 3) the first and the last steps completely debonded with 0.1 in. (2.5 mm) gaps at the beginning and the end of the joint and 0.01 inch (0.25 mm) gaps at all other step ends.

Numerical results for the stresses and forces are presented in Figs. 4-7. Figure 4 shows the stresses in the titanium layer as a function of the distance from the beginning of the joint (at step 1), It is seen that the flaws affect only the local stress distribution. The stress state in steps 3-6 are not significantly altered. In the vicinity of the flaws, i.e., near the beginning of step 2 and the end of step 7, significant change in the stresses are observed. This is because there is no load transferred from the titanium to the composite in the debonded region. In Fig. 5, the forces in the composite layer are given as a function of the distance from the beginning of the joint (at step 1). For the flawed configuration, load transfer takes place between the beginning of step 2 and the end of step 7. The slight discontinuity of the force for the case with end gaps at the step ends is attributed to the axial load carried by the adhesive at the gap. Figure 6 shows the stresses in the composite layer and Fig. 7 shows the shear stresses in the adhesive layer. Both figures show that only in steps 2 and 7 are the stress distributions significantly influenced by the flaws.

Conclusion

The problem of an adhesively bonded stepped-lap joint with bondline flaws has been analyzed. A numerical solution of the governing differential equations with appropriate boundary conditions was obtained. From the results presented, the following observations are made:

1) The presence of bondline flaws, either in the form of debonding or end gaps, or both, affects the stress distributions in both the adherends and the adhesive only in the vicinity of the flaws.

2) The presence of debonding causes an increase in the maximum shear stress in the adhesive layer near the debonded region.

3) The presence of the step-end gaps causes a slight discontinuity of the forces in the composite layer at the step ends. As a result, the shear stress in the adhesive layer is slightly higher at the step ends. However, there is virtually no effect on stress distribution in the joint.

4) Complete debonding of the first step causes no load transfer in that step, thus increasing the stress in plate 1 in the second step. Complete debonding of the last step causes no load transfer in that step, and thus increases the stresses in plate 2 in the seventh step.

5) The normal (or peel) stress at end gaps has been ignored in the present analysis. This stress may be significantly affecting the performance of a stepped-lap joint.

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